

## A Determination of the Three-Dimensional Seismic Properties of Anorthosite: Comparison Between Values Calculated From the Petrofabric and Direct Laboratory Measurements

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We have undertaken a comparative study of the seismic properties of an anorthosite. The seismic properties have been calculated from the lattice preferred orientation (LPO) of plagioclase (90% by volume) and olivine (10% by volume), and the elastic constants of these minerals. Laboratory measurements of  $V_p$  (24 directions) and  $V_s$  (6 directions) have been made on the same sample at room temperature and pressures up to 1.0 GPa. The two techniques agree with 0.1 km/s for all orientations studied when the Voigt averaging method is used. Hill and Reuss averaging methods give calculated velocities significantly lower than the experimental ones. A study of the difference between the velocity values given by the two methods reveals a systematically larger departure in orientations normal to the foliation. The orientation dependent difference between the two techniques can be explained if there are small errors in the elastic constants for single crystal plagioclase used in the initial calculations. The axial LPO symmetry of the plagioclase with all the crystallographic  $b$  axes parallel to the specimen symmetry axis allowed us to calculate  $C_{22}$  (normal to (010)) directly from the laboratory measurement of  $V_p$ . Some constraints can be put on  $C_{22}$ ,  $C_{44}$ ,  $C_{66}$ ,  $C_{12}$  and  $C_{23}$ . On average our data require 10% modification of the single crystal constants reported by Ryzhova (1964)

### INTRODUCTION

The deep continental crust reflection seismic profiles (BIRPS, COCORPS, ECORS, etc. ...) have provided much new data on the seismic properties of the lower crust. The interpretation of the seismic profiles relies heavily on the knowledge of the seismic properties at various pressures and temperatures attained from direct laboratory measurements of typical rock types [Christensen, 1965; Fountain, 1976; Kern and Richter, 1981]. Although the initial emphasis of the regional studies was on the compressional wave (P wave) velocities in two dimensional (2-D) profiles, more recent studies have used P wave and shear wave (S wave) recordings with cross-cutting profiles to elucidate the three dimensional (3-D) seismic properties of the lower crust. To constrain the interpretation of such three dimensional data, we need a knowledge of the 3-D seismic properties of rocks, for example, from direct laboratory measurement in many directions (more than three), which is extremely time consuming. Alternatively, the petrofabric method may be used. It may also be time consuming depending on the method (universal stage, X ray or neutron texture goniometry), crystal and specimen symmetry; but petrofabric data can be used to provide vital information about other anisotropic tensorial properties (e.g., electric and magnetic), as well as further the interpretation of the anisotropy of seismic properties. In particular, the petrofabric method provides a method to calculate the 21 elastic constants of a rock with triclinic specimen symmetry. Data in such a form is a particularly compact method of introducing anisotropic seismic (elastic) properties into three dimensional synthetic seismograms [Mainprice and Nicolas, 1989].

In this paper we compare the seismic properties ( $V_p$ ,  $V_{s1}$ ,  $V_{s2}$ ), of an anorthositic rock obtained by direct measurements at pressures up to 1 GPa, and values calculated from the lattice preferred orientation of plagioclase and olivine and their elastic constants. Modeling the seismic properties, with a simple assumption on the symmetry of the lattice preferred orientation, permits us to discuss the accuracy of the elastic constants available for single crystals of plagioclase [Ryzhova, 1964; Aleksandrov et al., 1974].

### PETROLOGY AND SPECIMEN STRUCTURE

The specimen studied is an anorthosite from the Precambrian Grenvillian (1100 Ma) basement in Oklahoma. It is probably representative of the many anorthosite bodies that are volumetrically important components of Precambrian cratons, themselves an important element of the lower continental crust [Ronov and Yowoshevsky, 1968]. In hand specimen the rock is black and almost monomineralic. A foliation is clearly defined by the laths of feldspar, but no lineation can be discerned. A reference frame was defined with Z normal to the foliation and X and Y at right angles in the foliation plane.

In thin section, the rock composition was determined by modal analysis to be 90% plagioclase and 10% olivine (Figure 1). A few grains of amphibole, clinopyroxene and biotite were also observed. The grain size is large (up to 1 cm) for all minerals, with no interstitial phases. The plagioclase grains are euhedral to sub euhedral. In sections parallel to the foliation (XY) no direction of preferred shape mineral alignment could be detected. In sections perpendicular to the foliation (XZ, YZ) they are tabular with surface areas up to 5 mm<sup>2</sup>. Both XZ and YZ sections have identical characteristics: plagioclase grains have a shape ratio, greater than 4 for more than 50% of the grains. The majority of grains are twinned with the albite law, we have also observed

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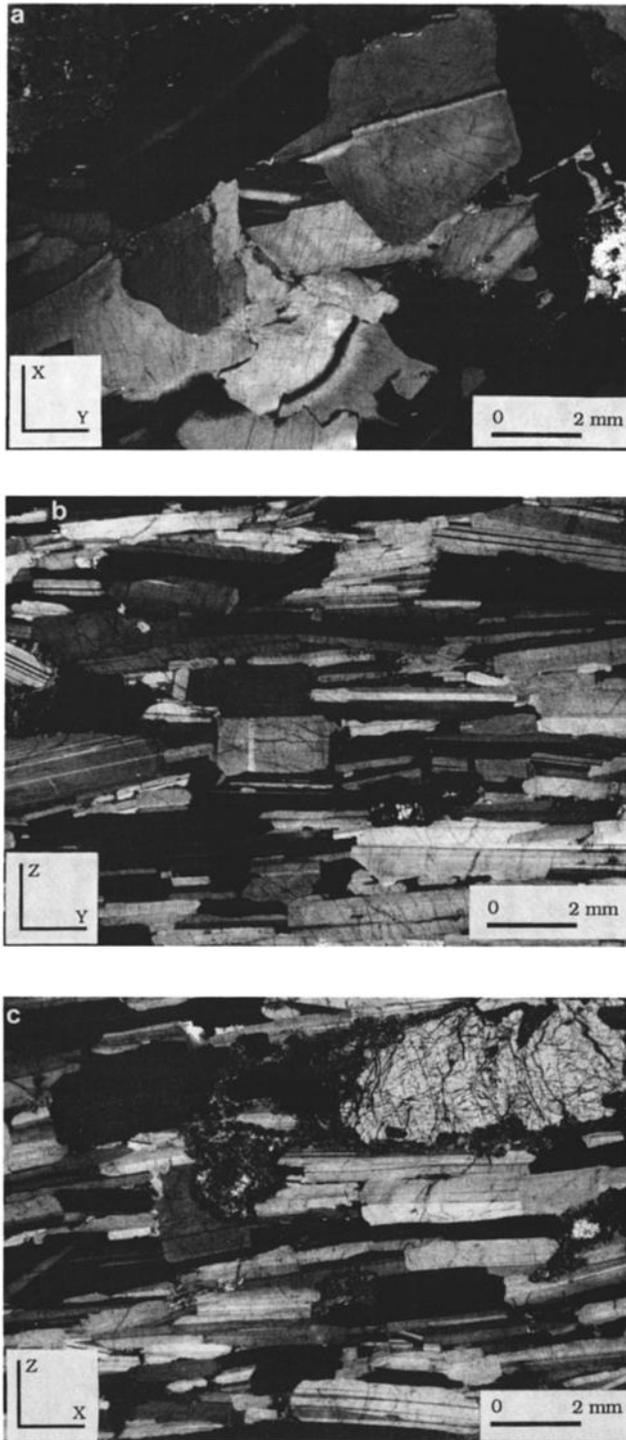


Fig. 1. Thin section of the Oklahoma anorthosite, (a) parallel to the foliation, (b) and (c) perpendicular to the foliation. No lineation could be detected. The majority of grains shows albite twins, and in a few cases pericline twins.

Carlsbad and less frequently pericline twins. The trace of the albite twin plane (010) in every thin section is systematically perpendicular to the Z reference direction (normal to foliation). Hence we can predict a strong plagioclase crystallographic fabric with (010) parallel to the foliation. The similarity of the microstructure in the XZ and YZ sections combined with the lack of mineral alignment in the foliation (XY section) suggests the rock has an axial symmetry around Z.

The twins in the plagioclase grains are parallel sided and transect the grains. No trace of undulatory extinction was detected in either the plagioclase or the olivine. The above features suggest that the rock is of igneous origin with no evidence of plastic deformation.

#### ORIENTATION MEASUREMENTS OF PLAGIOCLASE

##### Introduction

There are relatively few studies of the lattice preferred orientation (LPO) in plagioclase due to the time consuming nature of such measurements in triclinic minerals. Many more authors have measured the orientation of the optical axes; however these are insufficient to unambiguously deduce the crystallographic fabric. *Wenk et al.* [1986] introduced a convenient method for measuring the orientation of the crystallographic axes based on the work of *Burri et al.* [1967]. These authors tabulated from all the available data the angular relationships between the crystallographic directions and optic axes as a function of anorthite content on a stereogram (Figure 2). In all the figures the crystallographic notations are  $(h,k,l) \Leftrightarrow$  plane  $h,k,l$ ,  $+[h,k,l]$  or  $-[h,k,l] \Leftrightarrow$  direction  $h,k,l$ ,  $+\perp(h,k,l)$  or  $-\perp(h,k,l) \Leftrightarrow$  normal to  $(h,k,l)$  in a direction positive or negative. Note that  $[h,k,l]$  is not in general parallel to  $\perp(h,k,l)$  in the triclinic crystal symmetry. The method of *Wenk et al.* [1986] consists of measuring the orientation of the optic axes and the two cleavage planes, (010) and (001), with a universal stage. The measurements are then plotted on the stereogram devised by *Burri et al.* [1967] (also see *Emmons* [1943]) where only one crystallographic orientation out of four possible orientations can fit the angular constraints for a given composition. The method can only be applied to grains containing the two cleavages. *Ji and Mainprice* [1988] used the two deformations twins, albite and pericline, in a similar manner. The method is very time consuming and involves several rotations of the data that can produce errors, hence *Benn and Mainprice*

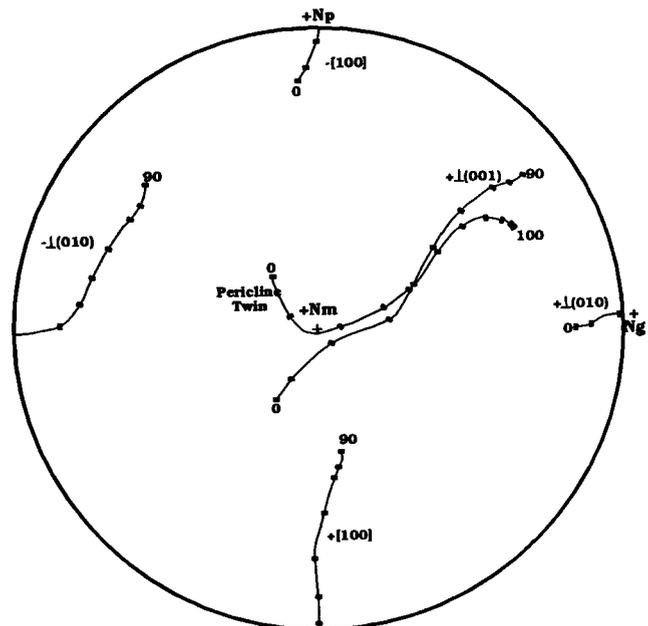


Fig. 2. This diagram (modified after *Burri et al.* [1967]) shows, in stereographic upper hemisphere projection, the angular relationships between crystallographic directions ( $[100]$ ,  $\perp(010)$ ,  $\perp(001)$ , pericline twin) and optic axes ( $N_p$ ,  $N_m$ ,  $N_g$ ), as a function of anorthite content from 0 to 90 or 100 percent. The 0 to 90 or 100 anorthite percent values are marked on the lines corresponding to the various crystallographic directions.

[1989] introduced an interactive computer program to facilitate the measurements.

The method has the important handicap that only grains in the section of observation containing two crystallographic planes (cleavages or twin planes) can be measured. Therefore the crystallographic fabric measured represents a sub fabric that is not necessarily representative of the total fabric. To overcome these difficulties, *Kruhl* [1987] proposed measuring on three perpendicular sections (XZ, YZ and XY). Grains that are not measurable in one section are statistically represented on the other sections. The sum of the measurements on the three sections should give a good estimate of the total fabric. *Ji and Mainprice* [1988] suggested measuring the optic axes of all the grains and comparing the orientation with those grains possessing two measurable crystallographic planes. In this manner some estimate can be made of the representativeness of the crystallographic measurements.

#### Method of Measurement

During the course of the present study we have modified the approach of *Ji and Mainprice* [1988], to the specific case of Ca rich plagioclase. The symmetry of the migration curves for crystallographic planes (cleavages (010) and (001) or twin planes albite or pericline) on the Burri stereogram makes the orientation determination non unique within experimental error ( $\pm 5 \text{ An}$ ) for anorthite contents An60-An90 (Figure 3). To overcome this problem it is necessary to determine the identity of the crystallographic planes. In fact, if the identity of the one crystallographic plane is known, the orientation is completely determined. When one is measuring the cleavages it is impossible to distinguish those planes which are in the same zone as (010) from (001) with the optical microscope. If we measure the albite and pericline twins we can distinguish between these planes by optical methods (Figure 4). The albite law is of the normal hemitropy type, the twin plane is the habit plane separating the individuals (e.g., (010)), and the rotation axis is normal to this plane. The twin plane (010) is therefore an optical symmetry plane: the host and twin lamellas show symmetric extinction about the twin plane [*Roubault et al.*, 1982]. The pericline law is of the parallel hemitropy type, the twin plane is irrational (rhombic section) and the rotation axis [010] lies in the rhombic section. The extinction is not symmetric. Using these extinction criteria it is possible to assign a crystallographic identity to either albite or pericline twins. The chances of mistaking the twin for another type are statistically remote given the very low frequency of other twin laws in plagioclase [*Roubault et al.*, 1982].

In summary, the method used here was undertaken in two stages.

1. We measured all the grains which contain two crystallographic planes (twins or cleavages) in three sections cut in the XY, XZ and YZ planes. A measurement is considered acceptable if the projections of two planes on the Burri stereogram coincide for the same anorthite content within 5%. If the composition indicated by the two planes falls outside the 5% anorthite error band then the measurement is repeated. After measuring all the grains containing two planes, we have a well-defined anorthite composition for the section, in the present example An  $68 \pm 2$ .
2. We measure all the grains which contain only one twin. The first operation is to identify the twin law, albite or pericline, by the method described above. It is easy to determine the correct orientation by plotting the four possible stereographic projections. The albite twin was much more common than the pericline, hence statistically grains either had both albite and pericline or only albite twins.

The universal stage measurements may be systematically in

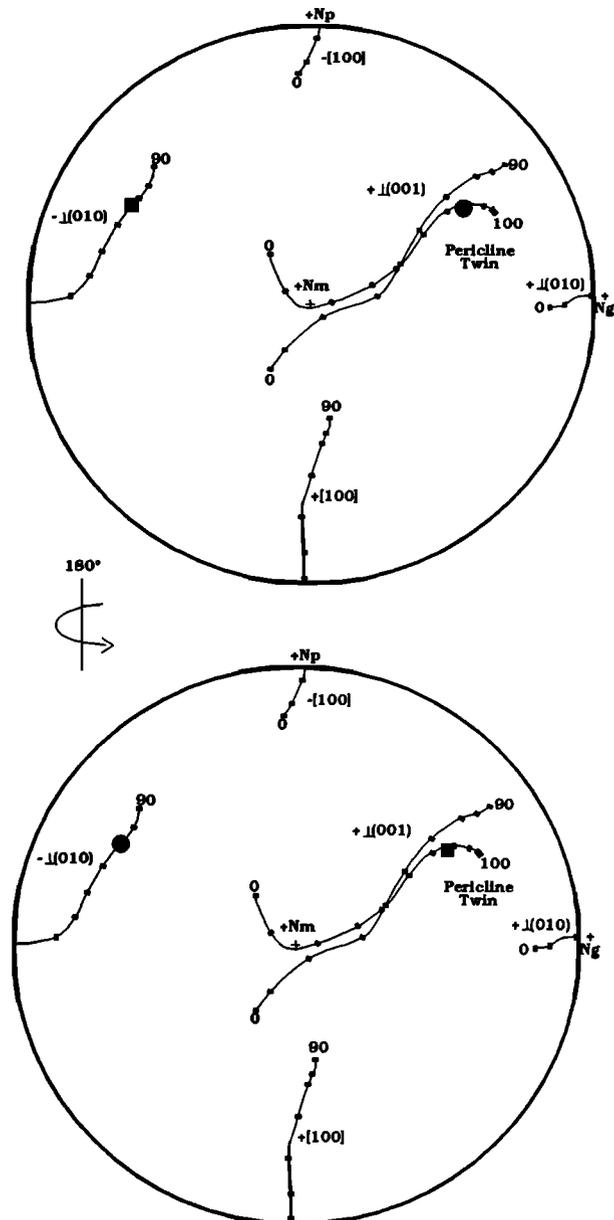


Fig. 3. For anorthite contents An60-An90, the migration curves for albite and pericline twins and for (010) and (001) cleavages are symmetric. The determination of the crystallographic orientation is non unique, and we must identify the crystallographic plane measured to overcome the problem (upper hemisphere stereographic projection).

error, and one should verify that the average plagioclase composition given by the measurements with two planes corresponds to that measured by an independent technique. In the present example electron microprobe analysis of plagioclase 15 grains gave An  $65 \pm 3$  within variation given by the optical measurements (An  $68 \pm 2$ ).

Using the above method we have avoided most of the problems encountered by the existing methods. We are able to measure more than 80% of the grains. However, we can only use this method if the majority of the grains are twinned, in particular at least 20% of the grains must have two twins for the determination of average composition. Alternatively, if an electron microprobe study indicates that plagioclase grain composition is homogeneous we can use this average composition together with the grains containing one twin for fabric measurement.

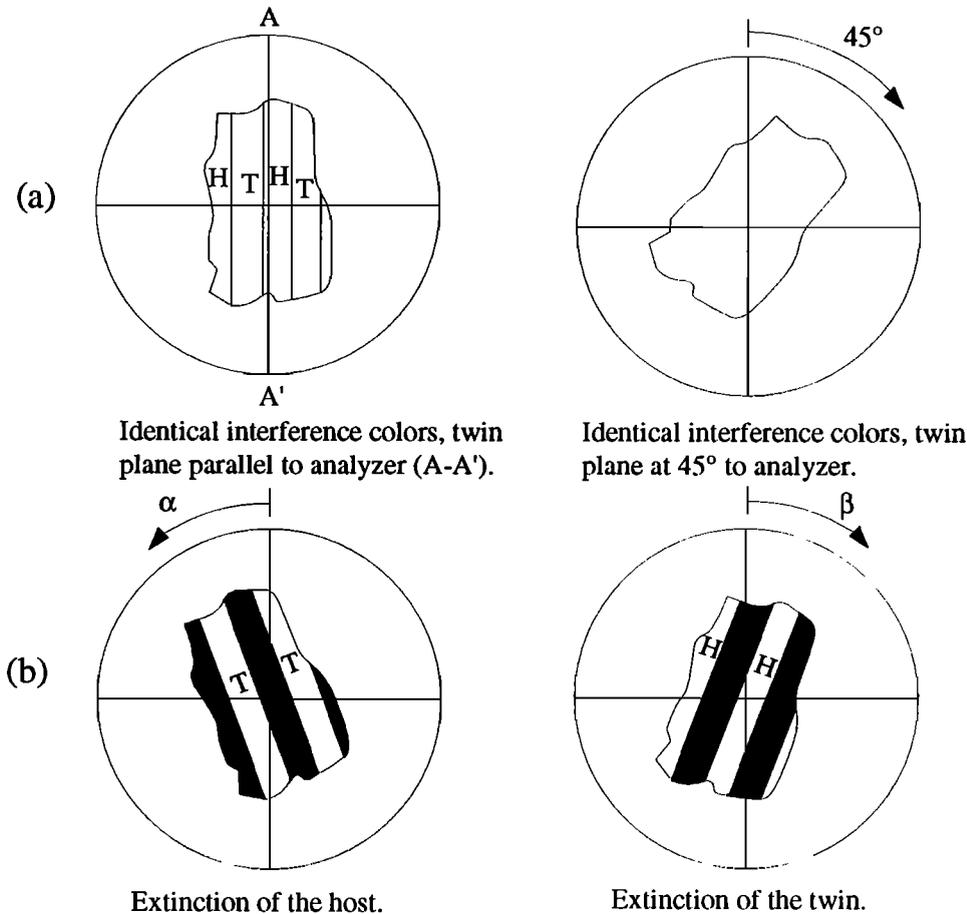


Fig. 4. Criteria for the identification of the albite twin:

(a) We verify that the twin plane is vertical (parallel to microscope axis): host (H) and twin (T) show the same interference color in the two positions (twin parallel to analyzer, we see the traces of the twin planes; twin at 45° to analyzer, we do not see the traces of the twin planes). (b) The extinction of the host and twin parts of the grain are symmetric  $\alpha = \beta$  [Roubault *et al.*, 1982], so the twin law is albite.

#### Lattice Preferred Orientation of the Anorthosite

Using the technique described above we have determined the LPO of the Oklahoma anorthosite. The measurements were undertaken on three perpendicular sections XZ, YZ and XY for the plagioclase and olivine. The results are shown in figure 5 for the sections XZ and YZ. For plagioclase we have measured 98 grains on the XZ section, 146 grains on the YZ section and none on XY. On XY it was impossible to make any measurements; the grain size is very large, there are very few twins (less than 10% of grains are twinned), and the twins planes are at high angle ( $> 60^\circ$ ) to the microscope axis, resulting in a poorly defined plane which is impossible to measure. Our aim was to measure an identical area on each section, systematically treating all the grains observed. In so doing we obtain a fabric that is representative of a volume element (defined by the three thin sections,  $32 \text{ cm}^3$ ) of the rock. The number of measurements we can undertake depends on the grain size and the twin frequency. Both these parameters are a function of the rock fabric, hence we have not tried to measure the same number of grains on each section. The positive and negative crystallographic axes have the same fabric. The positive axes are represented by solid squares whereas the negative axes are represented by open squares in Figure 5. In Figure 6 we present contoured diagrams for both sections (XZ, YZ) where the positive and negative axes are not distinguished. We can explain the

absence of measurable grains on XY by the nature of the fabric. In particular, the maximum of the (010) pole is parallel to Z and the albite twin plane (010) is statistically parallel to the foliation (XY plane). Therefore it is not possible to observe the albite twins in the XY section. The crystallographic fabrics (Figure 5) show the following characteristics: - the (010) pole forms a strong point maximum perpendicular to the foliation (Z); - the (001) pole and [100] direction each form a nearly perfect girdle in the foliation (XY); - the plagioclase has an axial crystallographic fabric; - the fabric is extremely strong with a maximum of 45.1% for the (010) pole.

As a further test of the fabric we cut an additional section at 45° to the foliation plane. All the albite twin planes (010) were parallel to the foliation plane, thus confirming that the absence of measurable grains in the XY (foliation) plane is not related to the measurement technique, but rather to the plagioclase fabric.

Because olivine constitutes 10% of the volume of the rock, it is important to measure its fabric for seismic velocity calculations. We studied two sections for each orientation (XZ, YZ, and XY). Because of the large grain size (~1 mm), we could find only 15 grains. The fabric is not strong as shown by the data presented in Figure 7. Optical properties of the olivine indicate the composition is  $\text{Fo } 90 \pm 3\%$ . Although limited in number, these data represent all the olivine in a volume of  $64 \text{ cm}^3$  of the specimen and hence are appropriate for seismic velocity calculations as discussed later.

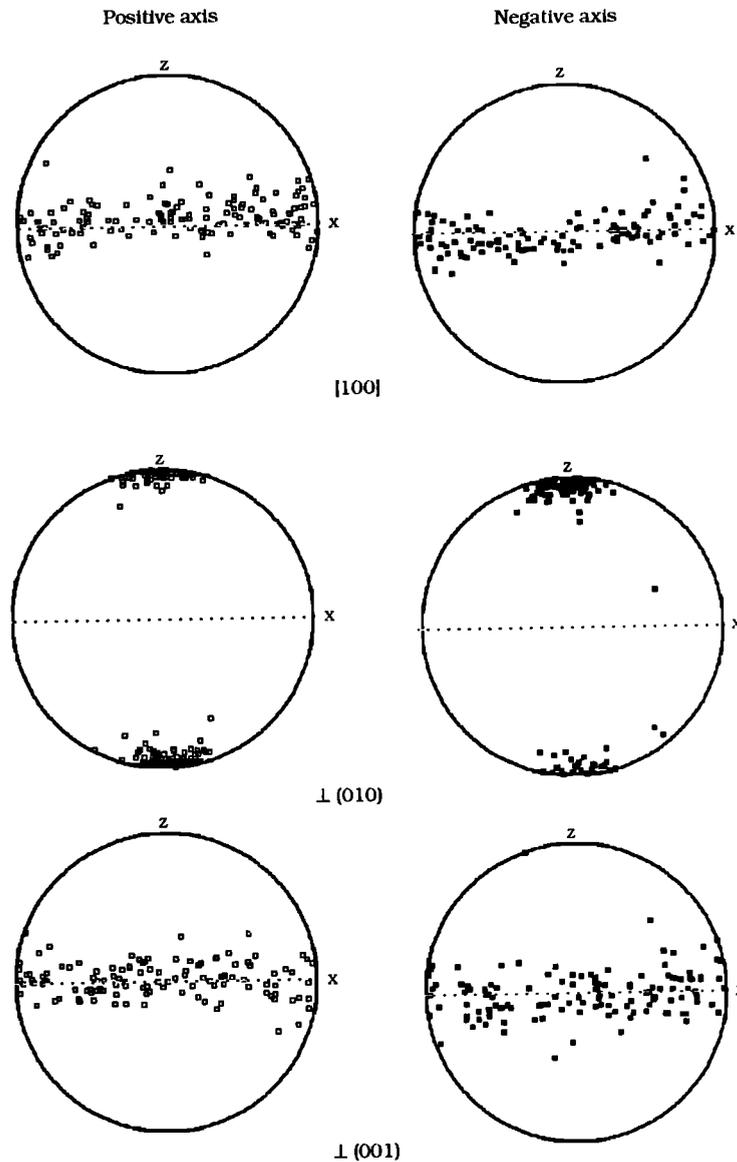


Fig. 5. Pole figures showing the distribution of positive and negative crystallographic axes for the plagioclase: [100], the normal to (010) and the normal to (001). The stereographic projection is in the lower hemisphere and is the same in all subsequent stereograms. Based on 244 measurements.

## SEISMIC PROPERTIES

### Laboratory Measurements

The seismic velocities were measured almost continuously between 0.02 and 1 GPa using the pulse transmission technique described in detail by *Christensen* [1985]. These measurements have been made for 24 directions for  $V_p$  and 6 directions for  $V_s$ . As the rock properties have axial symmetry we present velocity data for a few inclinations to the symmetry axis (which is normal to the foliation) (Figure 8; four orientations for  $V_p$  and three for  $V_s$ ). The curves are very similar to previous studies in that they show a steep increase in velocity in the 0.02 to 0.30 GPa pressure interval followed by an almost linear and weak increase in the 0.3 to 1.0 GPa pressure range. The steep increase at low pressure is attributed to the closure of micro cracks. Because the velocities are almost constant above 0.3 GPa, anisotropy does not vary with pressure. We can make the assumption that the elastic properties

of plagioclase are virtually independent of pressure in this pressure range. We will present in detail the velocities at 0.8 GPa, which is the pressure we have chosen to be representative of conditions in the lower continental crust where cracks will be closed in a dry specimen.

The results are presented in the form of a stereogram (Figure 9). The P wave velocity measurements in the 24 directions are presented in the same reference frame as used for the petrofabric measurements. The maximum  $V_p$ , 7.93 km/s, is perpendicular to the foliation (i.e., parallel to the Z axis). A zone of minimum  $V_p$  (6.83 km/s) forms a girdle at  $45^\circ$  to the foliation plane (XY). The  $V_p$  seismic anisotropy coefficient ( $A = (V_{\max} - V_{\min})/V_{\max}$ ) is high at 13.8 %. The velocity is high parallel to Z, whereas it is uniform in the foliation (XY) at 7.1 km/s. The S wave velocities measured in six directions are presented in Figure 10. We have not contoured the values because of the relatively few directions of measurement.

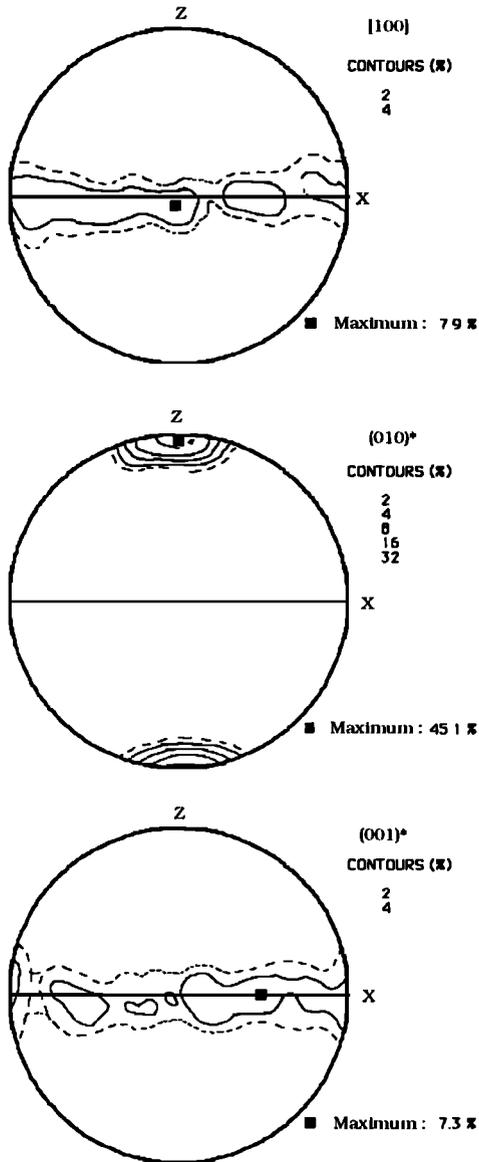


Fig. 6. Contoured pole figures of [100],  $\perp(010)$  and  $\pm \perp(001)$  of data from Figure 5. No distinction was made between positive and negative crystallographic directions.

*Seismic Properties Calculated From LPO*

The measurements of the petrofabrics of plagioclase and olivine provide the orientation data necessary for the calculation of the seismic properties of the anorthosite. We will use the plagioclase elastic constants given by Ryzhova [1964] for An57-60, which were measured at room pressure for the velocity calculations. A discussion of all the available elastic constants of plagioclase is given in the appendix. An57-60 is the closest published composition to the sample used here (An68). For olivine we use the constants and pressure derivatives given by Kumazawa and Anderson [1969] for 0.8 GPa hydrostatic pressure and room temperature.

The P wave velocity distribution has been calculated using the Christoffel equation from the plagioclase petrofabric data (Figure 11) and correspond to room pressure values as no pressure derivatives are available for the single elastic constants. We have calculated the elastic average using the three classical methods Voigt, Reuss and Hill. The results agree with the conclusions of

Crosson and Lin [1971] and Peselnick et al. [1974] that the Voigt average gives the closest approximation to the laboratory measurements, while the Reuss and Hill are significantly lower. Even the Voigt velocities are slower than the laboratory measurements; this is due to two factors: - It is important to take the olivine volume fraction into account in our calculations. The much higher elastic moduli and density of olivine result in faster seismic velocities in olivine than in plagioclase. - There is a possible effect of hydrostatic pressure on the elastic constants of plagioclase (see laboratory measurements above).

The result of the calculation for the Voigt average for a composition 90% plagioclase and 10% olivine is given in Figure 12. The maximum P wave velocity (7.8 km/s) is normal to the foliation, and the minimum (6.8 km/s) forms a girdle at 45° to the foliation plane (XY).

The P wave symmetry of the rock is axial, identical to the symmetry given by the plagioclase alone. The result is consistent with the P wave distribution of a plagioclase single crystal (Figure 13) where the fast direction is parallel to [010].

We have also calculated the distribution of the two polarized S waves velocities (Figure 14), where we have designated  $VS_1$  as faster than  $VS_2$ . The polarization planes have also been presented for S1 (Figure 14). The velocities and polarizations are consistent with the laboratory measurements. The S waves are polarized

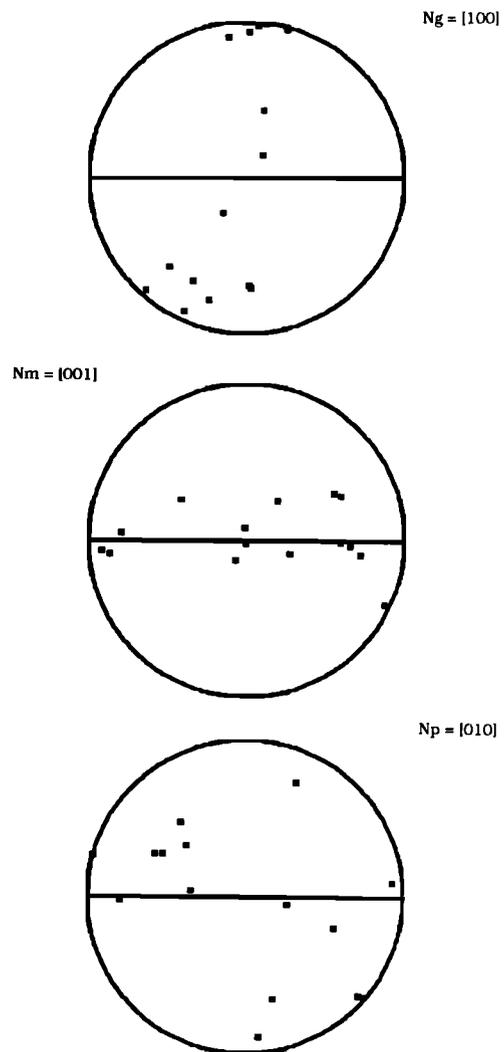


Fig. 7. Pole figures for [100], [010] and [001] for olivine. Based on 15 measurements.



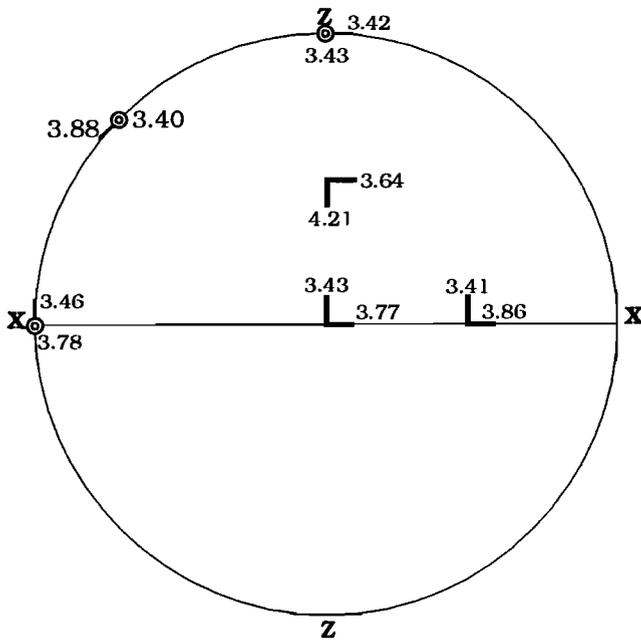


Fig. 10. Laboratory measurements of  $V_s$  (Kilometer per second). For each direction of propagation, the two lines represent the orientation of the polarization planes (direction of the particle motion).

either parallel to the foliation (XY) or normal to the foliation (parallel to X), which gives an axial symmetry.

*Modeling*

The seismic velocities that we have calculated from the petrofabric data are very close to the experimental values (within 0.1 km/s). We expected the calculated values to underestimate the experimental values because we suspected that at room pressure single crystal elastic constants may be slight underestimates due to the presence of open cracks or cleavages. However, the room pressure elastic constants give a very close approximation to high pressure experimental values using the Voigt average.

There may be several possible causes for the slight discrepancy between the experimental and calculated values: (1) the single crystal elastic constants were determined on a crystal of An57-60 whereas our specimen had a composition of An 68. (2) The averaging techniques we have used are only strictly applicable to equidimensional grains, this may result in a systematically higher value normal to the foliation and a lower value in the foliation [Humbert et al., 1981]. (3) The axial sample symmetry is higher than the single crystal symmetry of plagioclase. The average elastic constants of the aggregate will depend critically on a small number of the single crystal constants, whereas certain constants will not intervene in the average. The average will be more sensitive to the errors in these critical constants.

Because specimen symmetry is nearly perfectly axial, we introduce a model LPO with perfect axial symmetry to model the elastic properties. In using a perfect axial symmetry we eliminate the possible errors and imprecisions introduced by the U stage measurements. In the axial plagioclase model LPO the (010) pole is normal to the foliation in all the grains and [100] and the (001) pole are uniformly distributed in the foliation (XY) plane. We have created an axial fabric for olivine with [010] normal to the foliation (Z) and [001] uniformly distributed in the foliation, although we have no a priori control over its fabric. To verify if our model fabric is an appropriate model of the seismic properties of the real anorthosite we have calculated the P wave velocity

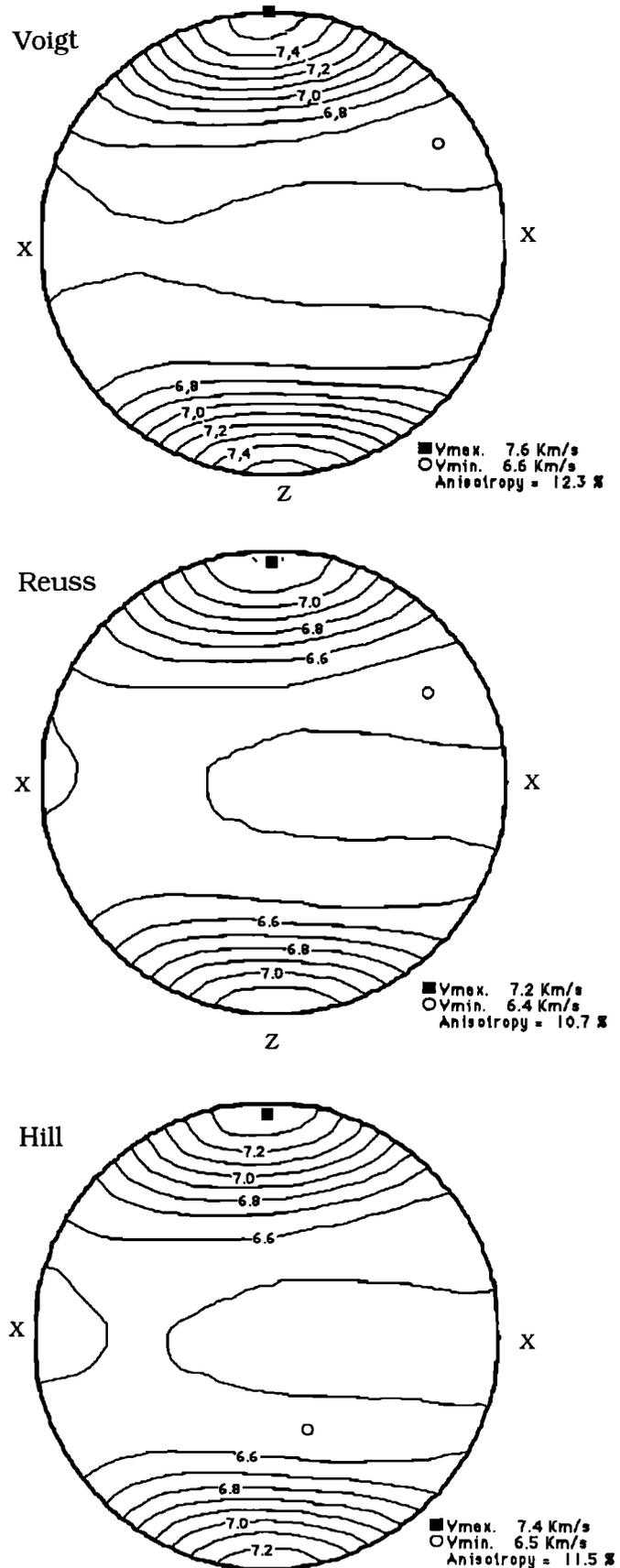
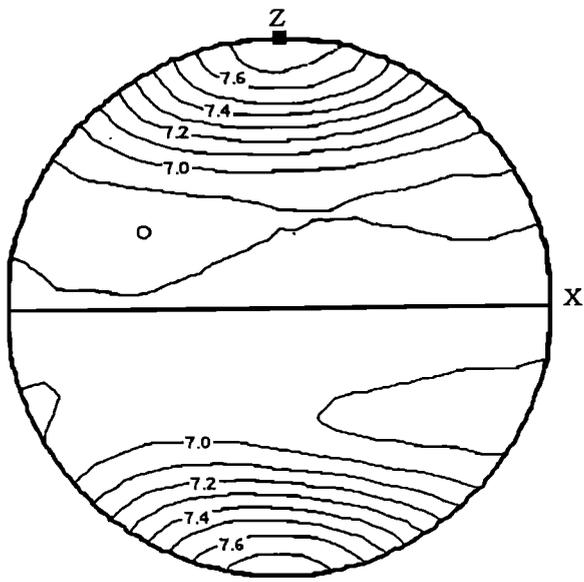


Fig. 11. The calculated Voigt, Reuss and Hill average,  $V_p$  (in Kilometer per second) for the Oklahoma anorthosite. Only the plagioclase is taken into account in these calculations. The Voigt average is in closest agreement with laboratory measurements.



■ Vmax. 7.8 Km/s  
 ○ Vmin. 6.8 Km/s  
 Anisotropy = 11.8 %

Fig. 12. Calculated Voigt average  $V_p$  for the anorthosite with a composition of 90% plagioclase ( $d=2.68 \times 10^3 \text{ kg m}^{-3}$ ) and 10% olivine ( $d=3.3 \times 10^3 \text{ kg m}^{-3}$ ).

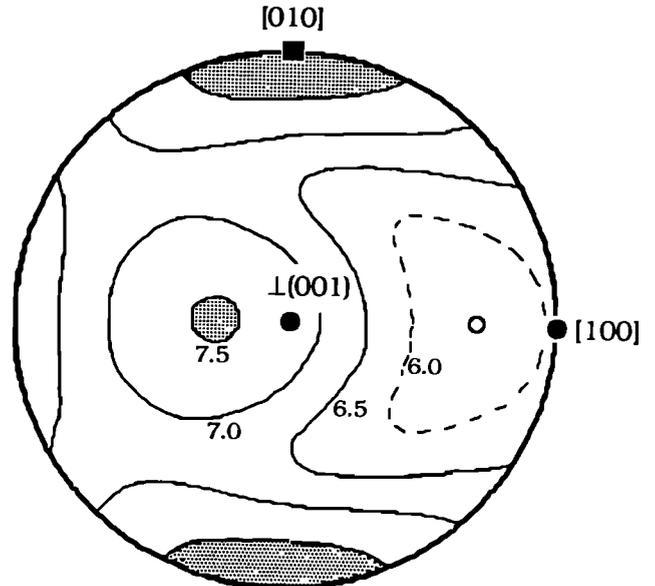
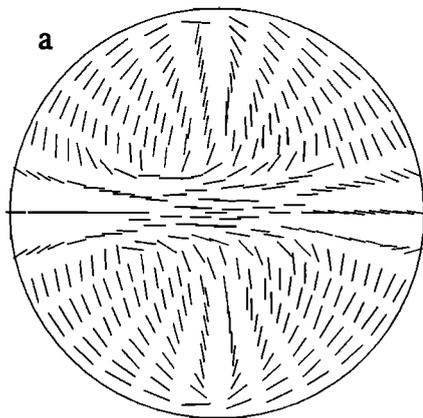
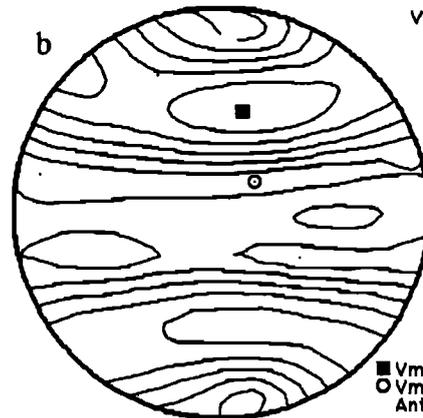


Fig. 13. The  $V_p$  of a single crystal of plagioclase An57. The elastic coefficients [Ryzhova, 1964] were calculated using a monoclinic crystal symmetry.

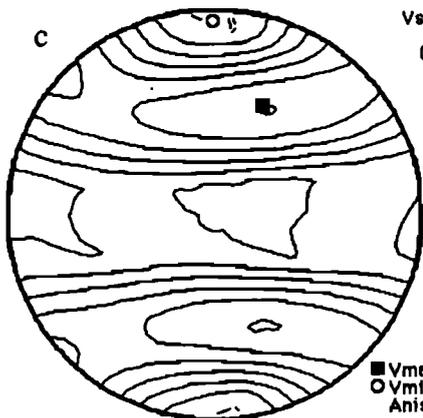


Polarisation



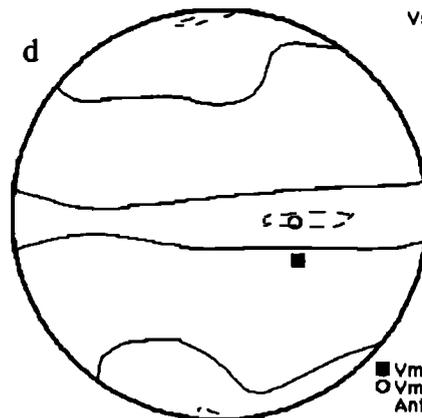
Vs1-Vs2  
 CONTOURS (Km/s)  
 0  
 0.1  
 0.2  
 0.3  
 0.4  
 0.5  
 0.6

■ Vmax. 0.6 Km/s  
 ○ Vmin. 0.0 Km/s  
 Anisotropy = 100.0 %



Vs1  
 CONTOURS (Km/s)  
 3.7  
 3.8  
 3.9  
 4.1  
 4.2  
 4.3

■ Vmax. 4.3 Km/s  
 ○ Vmin. 3.7 Km/s  
 Anisotropy = 14.4 %



Vs2  
 CONTOURS (Km/s)  
 3.6  
 3.7  
 3.8

■ Vmax. 3.8 Km/s  
 ○ Vmin. 3.6 Km/s  
 Anisotropy = 5.5 %

Fig. 14. The calculated S wave properties of the anorthosite.  $V_{s1}$  is arbitrary defined as faster than  $V_{s2}$ . Velocities in Kilometer per second.

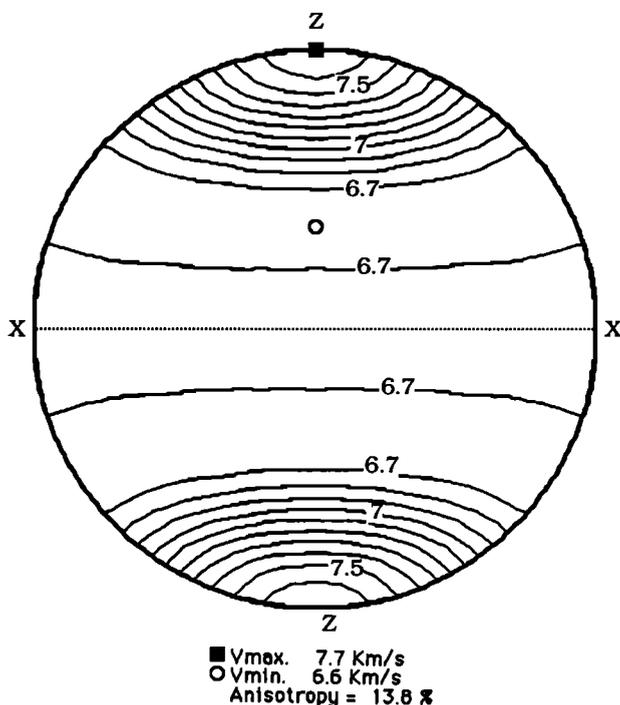


Fig. 15. P wave properties of a hypothetical aggregate with a composition 90% plagioclase and 10% olivine. The aggregate has a perfect axial symmetry with all the plagioclase b axes parallel to Z specimen axis (north).

(Figure 15). The calculated  $V_p$  diagram has identical symmetry to the laboratory measurements. The minimum and maximum  $V_p$  are the same in both cases. The P wave anisotropy ( $A=13.8\%$ ) for the theoretical model is greater than real aggregate, which is due to the perfect orientation of the crystals in the model. We can further appreciate the effectiveness of the axial model by comparing the model and experimental values in the plane normal to Z (Figure 16). The discrepancy between the model and experimental values is greatest near the XY plane. We will use the theoretical model in the calculations that assume perfect axial symmetry.

#### Elastic Constants

For seismic applications, it is useful to present the data in the form of contoured stereograms of P or S wave velocity. For a

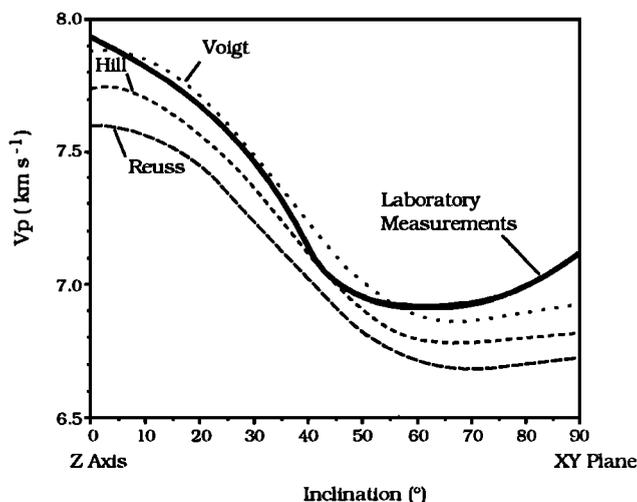


Fig. 16. Comparison between experimental and calculated P wave velocities projected on a plane in zone with the symmetry axis (Z). The Voigt, Hill and Reuss values are for the axial model aggregate. The curve representing laboratory measurements is an average for each angle between Z and XY.

quantitative comparison between LPO-calculated seismic data and laboratory velocity properties it is more appropriate to compare the aggregate elastic constants (in matrix form) derived by the two methods [Nye, 1972]. For the data derived from the LPO of the rock, the calculation of the Voigt average gives directly the 21 stiffness constants of the sample for a general triclinic symmetry.

To compare laboratory measurements with values for the perfectly axial symmetry case, we have calculated the stiffness constants for an aggregate with a theoretical axial LPO (see above). In the case of laboratory measurements we know that the elastic properties of a body (single or polycrystal) with axial symmetry are uniquely defined by five independent constants, elastic symmetry which is termed hexagonal in single crystal literature [Nye, 1972] or transverse-isotropic in the seismological literature [Christensen and Crosson, 1968]. From the laboratory measurements of P wave and S wave velocities in critical symmetry directions we can directly calculate the five elastic constants [Christensen and Crosson, 1968] as shown in Table 1. The results of the two independent calculations are given here as elastic stiffness matrices.

TABLE 1. Relationships Between Wave Velocity and the Elastic Constants of an Aggregate With Perfect Axial Symmetry

Wave Type	Propagation Direction (x3 Axis // to Symmetry Axis)	Direction of Particle Motion	Wave Velocity
Longitudinal	Z	Z	$[c_{11}/\rho]^{1/2}$
Transverse	Z	X (or Y)	$[c_{66}/\rho]^{1/2}$
Longitudinal	X (or Y)	X (or Y)	$[c_{33}/\rho]^{1/2}$
Transverse	X (or Y)	Z	$[c_{66}/\rho]^{1/2}$
Transverse	X (or Y)	$x_2$ (or X)	$[1/2(c_{33}-c_{23})/\rho]^{1/2}$
Quasi-longitudinal (high velocity)	45° from Z axis	not X, Y or Z	$\{[(c_{13}+c_{66})^2+(c_{33}-c_{11})^2]^{1/2} + 1/2(c_{33}+c_{11}+2c_{66})/\rho\}^{1/2}$

After Christensen and Crosson [1968]. Note axial symmetry axis is parallel to the Z sample axis.  
Relations between non-zero coefficients:  $c_{55} = c_{66}$ ,  $c_{12} = c_{13}$ ,  $c_{22} = c_{33}$ ,  $c_{44} = 1/2 (c_{33}-c_{23})$ .

162.65	046.90	046.90	000.00	000.00	000.00
046.90	138.16	054.47	000.00	000.00	000.00
046.90	054.47	138.16	000.00	000.00	000.00
000.00	000.00	000.00	041.84	000.00	000.00
000.00	000.00	000.00	000.00	033.64	000.00
000.00	000.00	000.00	000.00	000.00	033.64

168.53	040.68	040.68	000.00	000.00	000.00
040.68	135.86	057.23	000.00	000.00	000.00
040.68	057.23	135.86	000.00	000.00	000.00
000.00	000.00	000.00	039.31	000.00	000.00
000.00	000.00	000.00	000.00	031.35	000.00
000.00	000.00	000.00	000.00	000.00	031.35

(1) is calculated from the lattice preferred orientation, (2) is calculated from laboratory measurements assuming perfect axial symmetry. In the orientation of our reference frame, the elastic stiffnesses are  $c_{12} = c_{13}$ ,  $c_{22} = c_{33}$ ,  $c_{55} = c_{66}$ ,  $c_{44} = (c_{33} - c_{23})/2$  and the independent constants are  $c_{11}$ ,  $c_{12}$ ,  $c_{22}$ ,  $c_{44}$ ,  $c_{55}$ . We see that  $c_{11}$  is significantly higher for the laboratory measurements. However, the laboratory measurements of  $c_{12}$ ,  $c_{22}$ ,  $c_{44}$  and  $c_{55}$  are all lower than the LPO values.

Using the experimental velocity values and the LPO as reference data, we have attempted to vary the values of single crystal constants until a close agreement is found between experimental and calculated velocities. In undertaking such a calculation, it is clear that we cannot constrain all the elastic constants of the single crystal because of the axial specimen (rock) symmetry. The elastic properties of the specimen are defined by five independent elastic constants, hence we can only constrain five of the 13 single crystal constants.

Rather than use a general method of inversion we have tried to use direct modelling to understand the relationship between the individual single crystal constants and those of the aggregate. When we calculate the average properties of the aggregate, we perform a rotation of the single crystal elastic constant for each grain orientation, the relation between the single crystal and aggregate properties is a function of the LPO which defined the rotations. It is possible to use the equations of elasticity to formulate the relationships as a function of sample symmetry; however, the calculations are very complicated. An alternative approach is to use direct modeling to study the relationships between the single crystal and the aggregate. The single crystal constants are modified one by one and the resulting aggregate matrices are recorded (e.g., modifying  $C_{11}$  by 10%, calculating the aggregate properties and recording which aggregate constants are modified). We have imposed an arbitrary  $\pm 10\%$  variation for each of the single crystal constants, and made a complete series of calculations. This shows that single crystal constants  $C_{15}$ ,  $C_{25}$ ,  $C_{35}$  and  $C_{45}$  have no influence on the aggregate constants; this is due to the axial sample symmetry. The influences of the other constants on the aggregate are summarised in Table 2. In the first case (Table 2 case 1), we have seen that one aggregate constant is

a function of one single crystal constant, so we can directly recalculate this constant. In the other two cases (Table 2 cases 2 and 3), one aggregate constant depends on two single crystal constants (a variation on each of these two constants induces a variation on the aggregate constant), the problem is underdetermined. In last case (Table 3 case 4), two aggregate constants depend on four single crystal constants, the problem is again underdetermined. From our data on the specimen we can only rigorously determine one single crystal constant ( $C_{11}$ ). The constant  $C_{11}$  should be increased for the experimental and calculated values to match. If we make an additional hypothesis we can recalculate four addition constants. We have noticed that in relation (2) the single-crystal constants are all under evaluated. It may be that the error is small for one of the constants or large for the other, we have assumed that to a first approximation, the error is the same for both values, then in that limit the calculation is possible. The relation (3) is similar to (2), the calculation can be undertaken in the same limit. For the last two relationships (4), we cannot calculate the constants and we note that the same constants should be increased in relation (3) and decreased in relation (4).

Using a method of successive approximations, we have recalculated the five single crystal constants until the aggregate constants are identical to the values derived from the velocity measurements.  $C_{22}$  should be increased by 7.5%,  $C_{44}$  and  $C_{66}$  reduced by 8.5%, and  $C_{12}$  and  $C_{23}$  reduced by 15%. We need to modify the constants by, on average, 10%. The change is larger than the revision of the original of Ryzhova [1964] given by Aleksandrov et al. [1974] (see appendix). In fact, our revised values agree better with the original values of Ryzhova [1964] than those of Aleksandrov et al. [1974]. The new values can only be applied with confidence to our sample geometry. We have only modified five constants out of the 13 for the monoclinic approximation for twinned plagioclase. It is probable that the error is of the same order for the other eight constants.

CONCLUSION

The comparison between calculated Voigt average seismic velocities and laboratory values shows that they are in good agreement (0.1 km/s), even for an aggregate of a crystallographically complex silicate such as plagioclase. The calculated values are certainly of high enough quality to characterize the three-dimensional seismic properties of rocks, for example, for comparison with seismic reflection profiles. The effect of pressure up to 1.0 GPa is also seen to have very little effect on anorthosite. Although, in detail, the single crystal constants that are available in the literature [Ryzhova, 1964; Aleksandrov et al., 1974] appear to be slightly in error ( $\pm 10\%$ ), it is clear they are sufficiently precise for many seismic applications. However, perhaps for other applications, such as thermodynamic calculations, the errors may have more important implications.

TABLE 2. The Four Possible Relationships Between the Single Crystal Elastic Constants and the Elastic Constants of an Aggregate with Perfect Axial Symmetry

Case	Elastic Constant of Aggregate	Elastic Constant of Single Crystal	Comment
(1)	$c_{33}$	$c_{22}$	one equation, one unknown
(2)	$c_{13}$	$c_{12}$ $c_{23}$	one equation, two unknowns
(3)	$c_{44}$	$c_{44}$ $c_{66}$	one equation, two unknowns
(4)	$c_{11}$	$c_{11}$ $c_{33}$ $c_{55}$ $c_{13}$	system of two equations,
	$c_{12}$	$c_{11}$ $c_{33}$ $c_{55}$ $c_{13}$	four unknowns

The aggregate constants are in the specimen reference (X, Y, Z) and the single-crystal constants in the reference ( $X_1=a^*$ ,  $X_3=c$ ).

## APPENDIX: ELASTIC CONSTANTS OF PLAGIOCLASE

To our knowledge, the only systematic study of the elastic constants of single crystals of the plagioclase solid solution series is that of *Ryzhova* [1964]. In the original publication the calculation of the elastic constants from the seismic velocities was done by hand. *Aleksandrov et al.* [1974] used the data of *Ryzhova*

[1964] to produce revised values using a computer routine. In Figure A1 we present the values of the elastic constants given by *Aleksandrov et al.* [1974] as a function of composition. In general, there is a linear trend with an increase of compliance with anorthite (An%) content for the diagonal values of the elastic matrix ( $C_{11}$ ,  $C_{22}$ , etc.). The diagonal terms are closely grouped for similar compositions An53, An56, and An 57-60. The off-

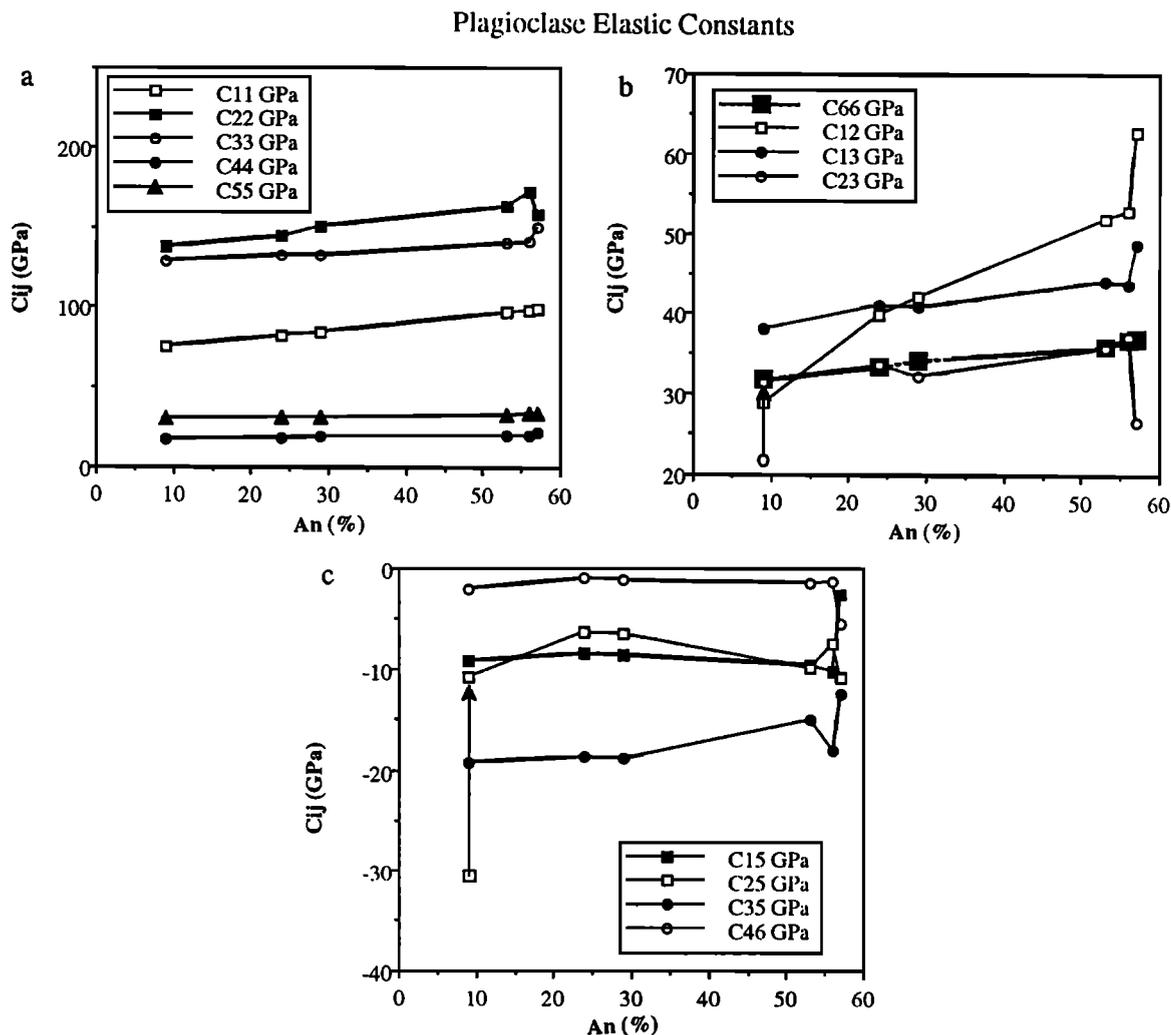


Fig. A1. Plagioclase elastic constants as a function of Anorthite mole percent in Gigapascal. Original values of  $C_{23}$  and  $C_{25}$  quoted by *Aleksandrov et al.* [1974] replaced by values indicated by arrowhead (see text).

TABLE A1. Elastic constants in GPa of plagioclase series derived from least-squares fit

	An0 2.62 g/cm <sup>3</sup>	An20 2.65 g/cm <sup>3</sup>	An40 2.68 g/cm <sup>3</sup>	An60 2.70 g/cm <sup>3</sup>	An80 2.73 g/cm <sup>3</sup>	An100 2.76 g/cm <sup>3</sup>
C11	69.85	80.13	90.40	100.67	110.94	121.21
C22	130.00	143.92	157.84	171.76	185.68	199.60
C33	125.77	131.32	136.87	142.42	147.97	153.52
C44	16.78	18.08	19.38	20.67	21.97	23.27
C55	29.22	30.85	32.47	34.09	35.72	37.34
C66	31.05	33.05	35.06	37.07	39.07	41.08
C12	26.48	36.21	45.94	55.67	65.40	75.13
C13	37.49	39.86	42.23	44.59	46.96	49.33
C23	30.24	32.49	34.73	36.98	39.22	41.47
C15	-8.26	-8.76	-9.26	-9.76	-10.26	-10.76
C25	-8.61	-8.34	-8.06	-7.78	-7.50	-7.23
C35	-20.04	-18.81	-17.58	-16.35	-15.13	-13.90
C46	-1.68	-1.51	-1.33	-1.15	-0.97	-0.79

Derived from linear least squares fit to the  $C_{ij}$  values of *Aleksandrov et al.* [1974] with  $C_{23}$  and  $C_{25}$  adjusted as discussed in the text.

diagonal terms ( $C_{ij}$ ;  $i \neq j$ ) such as  $C_{12}$  reveal that An 57-60 values are systematically different to An53 and An56 by an amount which is well outside the compositional variation indicated by the other samples. The other major anomaly to the general trend are the values of  $C_{25}$  and  $C_{23}$  for An9 which are -30.7 and 21.5 GPa, very different from the values of -10.4 and 32.6 GPa given by Ryzhova [1964].

We have taken the data of Aleksandrov et al. [1974] for which exact compositions are given (An9, 24, 29, 53 and 56) and made a linear least squares fit for each  $C_{ij}$  as a function of composition. In doing so we have excluded An 57-60 because of its anomalous behavior cited above. The values of  $C_{25}$  and  $C_{23}$  for An9 given by Aleksandrov et al. [1974] appear to be typing errors as they are so different from the original values given by Ryzhova [1964]. We suggest for  $C_{25} = -30.7$  should be -10.7 similar to the 1964 value of -10.4 and that  $C_{33} = 21.5$  should be 31.5 similar to the 1964 value of 32.6. None of the other  $C_{ij}$  values have changed significantly between the 1964 and 1974 tabulations. The least squares values of the elastic constants for various compositions of plagioclase are given in Table A1 together with the densities from synthetic plagioclase crystals of Kroll reported by Smith and Brown [1988].

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