Modelling the stochastic component of geophysical downhole measurements using scaling process.
Extraction of new attributes by Continuous Wavelet Transform.

Abstract

We propose a scaling process model from borehole measurements to describe the spatial variation of several geophysical important variables. The key argument that well logs exhibit a scaling behaviour is the absence of a characteristic length scale. The use of a Continuous Wavelet Transform (CWT) seems to be a particularly well adapted tool to systematically and reliably extract all the information about the singular part of the signal. Starting from synthetic fractal models, we show the influence of the various statistical parameters, in particular the number of Hurst, which expresses the local regularity of the medium. The fractals provide a new conceptual framework to draw part of the stochastic component from these signals, information too often ignored. The present work refines the analysis by investigating the behaviour of the fluctuations logs displaying variable local regularity. Indeed, these local spectral features of the scale laws which govern these signals are as many new attributes likely to optimize the characterization of the geological formations. These results open the way to a theoretical analysis based on local spectral exponents, and raises several questions which concern the use of signals regularity and its link to geological medium.

Keywords: stochastic processes, spectral exponent, wavelet transform, borehole, geophysics.

1. Introduction

The study of a signal is generally approached by searching for scales characteristic (in space, in time, in the space of the measurement and/or that of the frequencies). Many actual signals present details at all scales. In other words, they do not have any characteristic scale, but, on the contrary, they contain all scales simultaneously. The signals having this property are commonly named “process with scale invariance” and are governed by scale laws. In the field of geophysics in drilling, one checks that it is generally the case in log measurements. The scaling processes provide a new conceptual framework to account for the stochastic component from these signals.

The characterisation of facies using well logs to reveal some statistical properties have been extensively used to define the nature of the geological areas. It has been known that the small scale variations in geophysical boreholes data reflect the complexity of geological heterogeneities. In situ measurements of the upper crust petrophysical properties from deep scientific boreholes have suggested that well logs recording behave as a self affine process [1], [2], [3], [4]. The use of scaling process to describe well logs was presented by several studies performed in the last years [5], [2], [3]. The key argument that well logs exhibit a scaling behaviour is the absence of a characteristic length scale, and therefore they can be described suitably by a scaling process model.

A self affine process is usefully described by a set of spectral exponents and is usually referred to as the Hurst or roughness exponent estimated by the fit line slope of the power spectra in log-log plots. Knowing these exponents makes it possible to control the asymptotic statistical properties of the structure.

Because the sedimentary processes vary with time, geological sequences and its related petrophysical properties are non stationary, hence, the Fourier analysis is not the appropriate method to locate the spatial heterogeneity. Wavelet based estimators have been used very successfully for estimating scaling behaviour [3], [6], [7]. The central properties of self affine process enable one to establish a scaling relation between wavelet coefficients at a different scale from which the spectral exponent can be extracted.
2. Scaling process

The well recordings are governed by scale laws [5] where the power spectra are approximately inversely proportional to the wavenumber:

\[ S(k) \approx k^{-\beta}, \]  

(1)

where the global spectral exponent \( \beta = 2H + 1 \) determines the magnitude of short and long term correlations. Three special cases may be defined.

Specifically, \( \beta = 0 \) results in a white noise process, \( \beta = 1 \) gives a Joseph noise, while \( \beta = 2 \) defines Brownian noise. For \( 0 < \beta < 1 \), the process is called Gaussian fractal noise, and for \( 1 < \beta < 3 \), the process is not stationary and corresponds to a fractional Brownian motion.

The fractional Brownian motion (fBm) is the most popular model that displays scaling behaviour, used to describe well recordings [5], [8], [9], [7]. Introduced by [10], the fBm is a family of Gaussian processes \( \{B_h(z), z > 0\} \) indexed by a single parameter \( H \) called Hurst’s parameter \( (0 < H < 1) \) with zero mean, stationary increments and \( B_h(0) = 0 \). Its covariance is given by:

\[ E[B_h(z)B_h(n)] = \sigma^2 \left[ \frac{1}{2} \left| z \right|^{2H} + \left| z + n \right|^{2H} + \left| z - n \right|^{2H} \right] \]  

(2)

Note that \( H = 1/2 \), \( B_{1/2}(z) \) corresponds to the classical Brownian motion with independent increments. One of the main assets of fBm’s, is that they obey the self affine relationship:

\[ B_h(\lambda z) \approx \lambda^H B_h(z) \]  

(3)

Here the symbol \( \approx \) stands for statistical equality, and \( \lambda \in R^+ \). Eq. (3) expresses the fact that in self affine processes one must rescale the horizontal and vertical directions differently (\( \lambda \) for \( z \) and \( \lambda^H \) for \( B_h(z) \)) in order to have statistical invariance. Thus, self affine processes are anisotropic by construction in the horizontal and vertical direction. When \( H = 1 \), the process is self similar.

3. Wavelet analysis of scaling processes

The wavelet transform is a particularly well adapted tool for analysing scaling processes. Let us briefly review some of the important properties of wavelets. The Continuous Wavelet Transform (CWT) of a function \( s(z) \) is given by [11] as:

\[ C_{\omega}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(z) \psi_a(z) dz, \]  

(4)

each family test function is derived from a single function \( \psi(z) \) defined to as the analyzing wavelet according to [12] :

\[ \psi_a(z) = \psi \left( \frac{z - b}{a} \right), \]  

(5)

where \( a \in R^+ \) is a scale parameter and \( b \in R \) is the translation (\( \bar{z} \) is for the complex conjugate).

Indeed, the scale invariance can be reflected by CWT as long as the analyzing wavelet decreases quickly enough to zero [13], [14]:

\[ C_{\omega}(\lambda k, z_0 + \lambda z) \approx \lambda^{-H(\omega) - 1/2} C_{\omega}(k, z_0 + z) \]  

(6)

The Time/Scale map (or scalogram) is defined by the square of the wavelet coefficients:

\[ P(k, z) = \left| C_{\omega}(k, z) \right|^2 \]  

(7)

For sufficiently large wavenumbers, this relation can be re-written as [15]:

\[ P(k, z) \approx k^{-\beta} \]  

(8)

where

\[ \beta(z) = 2H(z) + 1 \]  

(9)

describes the local changes of the power law. When computed at every point of the signal, it defines a new statistical attribute that is called “pseudo-log”. It can be seen as a measurement of the strength of the singularity behaviour of the signal \( s(z) \) around a given point \( z \). The higher the exponent \( \beta(z) \), the more regular the signal \( s(z) \).

For a given wavenumber \( k \), the average of all the wavelet coefficients of \( P(k, z) \) spectrum over the total depth \( z \), determines an average wavelet power spectrum \( P_\beta(k) \):

\[ P_\beta(k) = \frac{1}{Z} \int_{-\infty}^{\infty} P(k, z) dz \]  

(10)

which can be related to the \( P(k) \) global power spectrum using the CWT energy conservation. Both spectra will exhibit at small scales or at large wavenumbers the same slope. In this case, the average wavelet power spectrum can be expressed by a power law:

\[ P_{\beta}(k) \approx k^{-\beta}, \]  

(11)

where \( \beta_0 \) is called the average spectral exponent.

4. APPLICATION

4.1 Synthetic data set

The \( s(z) \) stochastic part of a signal, also called fractional fluctuation, can be modelled with the Von Karman self-correlation function that characterises a family of self-affine stochastic processes at scales smaller than the correlation length [4].

In previous works [16], [17] a detailed study, based on numerical analysis on fractional Brownian motion (fBm), is carried out. It is shown how the local exponent, linked to the Hurst parameter \( H \), is used to quantify the heterogeneity contributions to the signal. The quality of a given analysing method is assessed by the difference between the spectral exponent, more precisely the Hurst number \( H \) chosen for the model and the estimated value from the analysis.

<table>
<thead>
<tr>
<th>Layers</th>
<th>( N )</th>
<th>Depth (m)</th>
<th>( \beta )</th>
<th>( \beta_0 )</th>
<th>( z_1 ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>512</td>
<td>86.88</td>
<td>1.2</td>
<td>1.11 ± 0.09</td>
<td>28.36</td>
</tr>
<tr>
<td>C2</td>
<td>256</td>
<td>125.89</td>
<td>2.3</td>
<td>2.5 ± 0.12</td>
<td>109.90</td>
</tr>
<tr>
<td>C3</td>
<td>1024</td>
<td>281.95</td>
<td>1.17</td>
<td>1.5 ± 0.13</td>
<td>203.92</td>
</tr>
<tr>
<td>C4</td>
<td>256</td>
<td>320.96</td>
<td>2.7</td>
<td>2.8 ± 0.13</td>
<td>301.46</td>
</tr>
</tbody>
</table>

In order to illustrate the potential of this approach, we present an academic model made of four layers which are distinct by their number of samples, and by their
fractal dimensions expressed in term of spectral exponent of the power law (table 1).

Fig. 1 shows the fluctuations $s(z)$ of this synthetic signal. The scalogram (Fig. 1(b)) reflects the energy distribution of the signal in the depth - wavenumber plane. The resolution decreases with the number of samples. If the sampling step of this academic example was 0.1524 m as for the many current log data, then, our CWT method would permit to detect a layer of about 5 m thick [17].

Fig. 1. (a) Synthetic fluctuations $s(z)$ of simulated geological model. (b) The scalogram $P(k, z)$ of synthetic signal $s(z)$ in (a). (c) Local changes of the spectral exponent $\beta(z)$ obtained from the signal displayed in (a). Note that the various singularities delineate the four layers of signal $s(z)$.

In this example, we note that the maximum energy value in the vicinity of logarithm wavenumber $k$ is equal to $-2.5$. This result is confirmed by the four local spectra $P(k, z)$ represented in Fig. 2. Indeed, from log $k = -2.5$, Figs 2(a), (b), (c) and (d) show clearly that the local spectra follow the decreasing power law described in section 3. We observe that there is coherence (see Table 1) between the predefined $\beta$ and estimated exponent values $\beta_l$ of the local power spectra $p(k, z)$ taken from the scalogram $P(k, z)$ at four different depths for the signal in Fig 1(a).

Note also, that the value of the average spectral exponent determined by CWT: $\beta = 1.42 \pm 0.05$ (Fig. 2(f)) is very close to the global exponent obtained with FT: $\beta = 1.45 \pm 0.04$ (Fig. 2(e)). This expected result is linked to the high number of vanishing moments of the Morlet wavelet. The CWT enables one to determine not only an average spectral exponent, but also a local spectral exponent at each sampling point of the signal. Fig. 1(c) shows local changes of the spectral exponent.

At any depth $z$, the power law exponent $\beta(z)$ fluctuates around $\beta$ the estimate value of slope $\beta$ derived from the power law spectrum. Comparison between the model in Fig. 1(a) and the evolution of the corresponding depth spectral exponent in Fig. 1(c), leads us to point out that the transition between two adjacent layers is marked by abrupt changes in the spectral exponent value. The various singularities delineate perfectly the four layers.

Fig. 2. Power spectra displaying a scaling behaviour of synthetic model $s(z)$ in Fig 1. (a) to (d). CWT local power spectra $\beta(k, z_l)$ can be compared to those of the model in Table 1. (e) CWT average power spectrum $P_{av}(k)$. (f) Global power spectrum $P(k)$. Comparison between the both spectra slopes (e) and (f) indicates very close values. The straight lines are the linear least squares fitting of estimating $\beta$.

4.2 Real data set

4.2.1 Geological setting

This numerical modelling allows to verify the ability of the CWT transform to extract new information from the stochastic component of a signal. We can now apply this method to real data of the MAR203 borehole drilled by the French National Agency for Radioactive Waste Management (Andra) [18]. This borehole is located near Marcoule in the Rhône valley (Fig. 3). This well samples from the Lower Aptian to Cenomanian sedimentary deposits stacked on the Western part of the Tethys passive margin on the Paleozoic crystalline basement located between the Massif Central and the Alps.

The Urgonian marine limestones are directly covered by Upper Albian and Cenomanian silstones and sandstones The Gargasian marl series, well known in the region, is absent below MAR203. Then, the platform gets immerse and is stacked with a thick Albian silty layer called "Couche Silteuse de Marcoule", (CSM). Because the basin remains shallow, the upper part of the CSM is strongly bioturbated. The porosity of this clayed-silty unit has been reduced considerably by a carbonate cementation. The thickness of the CSM ranges from 157 to 404 meters in MAR501 and MAR203 boreholes
respectively. During the Lower Cenomanian, a regression follows up at the top of the CSM with alternation of playa sandstones and lacustrine units. A lithological description of the sedimentary sequences is given in [19] and [20]. A review of an integrated geological sequential interpretation of these geological records can be found in [21].

Fig. 3. Simplified geological map and localisation of Andra wells on the SE Basin of France. (Map adapted from [18]).

4.2.2 Real data sets

According to this preliminary but conclusive check, it is possible to obtain the variation of the spectral exponent $\beta(z)$ for the following well logs: SFLU, NPHI, v-DT and GR of this borehole [9]. In this previous work, using a clustering neural network method as in [22] we have inferred the lithology sampled by the drill in two different ways: direct measurements and using their corresponding spectral exponents. It is shown in [9], that spectral exponents allows the determination of the lithology at least as well as the logs themselves. To pursue these investigations further, we explore here the petrophysical internal structur e of CSM silty unit using only the three different electrical resistivity measurements recorded in MAR203 borehole.

Two of them are measured by induction (ILM and ILD), the third by a current spherically focused (SFLU). The physical principles of all these electrical measurement devices are based on electromagnetism laws [23]. The resistivity logs are generally used to identify the pore fluid type and to measure the water saturation.

Several petrophysical experimental works [24], [25] show that the silty unit is characterized by a very low porosity (8%) and particularly by pores of very small sizes (75Å) that do not allow the migration of the fluids. The permeability of this formation is almost null. Taking into account these petrophysical characteristics, the flushing fluid cannot invade the porous space of the formation.

Indeed Figures 4(a) and (b) show that plots of the ILM, ILD and SFLU resistivities of the CSM are nearly superimposed. Resistivity of mud being constant and known (3.4 $\Omega$.m) progressive increase according to the depth of the three measured resistivities (Fig. 4 (a)) allows to evaluate the conductivity gradient of rock resulting of the compaction and the cementation of the silty unit.

For each log, three stochastic components of electric logs power spectrum and spectral exponent were calculated. Power spectra indicate a scaling behaviour with an exponent mostly falling in the range 1.4 < $\beta_z$ < 1.7 as illustrated by the various diagrams in Figure 7. This result is confirmed by the three histograms (Fig. 5) showing that the statistical distribution of local spectral exponents vary in the interval [1- 3] as expected from section 2.

These results are supported by an analysis in principal components carried out jointly to the three measures and on their respective spectral exponent allow to show that the two sets of measurements are anti-correlated as illustrated by Figure 6. Statistical analysis confirm that spectral exponents bear different information complementary to measurements. Pseudo logs are as many additional attributes making it possible "to burst" information in the first factorial plane and to reveal the relevant local structures contained in signal transients. We exploit these new statistical attributes to determine the internal structure of the CSM unit using CEM algorithm [26]. The clustering treatment of pseudo logs (Fig. 7 (c)) shows the existence of several subunits within the silty layer.

Between 470 and 770 m, the CSM remains on a massively silty and homogeneous level. The statistical extraction and analysis of spectral exponent enables this layer to be segmented into four subunits.
This progressive transition is located between 560 and 580 m below the surface. It had not been previously detected neither by logs data processing nor by core description. Nevertheless this transition corresponds to the most important variations of spectral exponents. We notice that 580 m corresponds exactly to the reversal of the gradient of salinity of the residual fluid formation resulting of the presence at the top and at the bottom of the CSM of two permanent water reservoirs.

Better than that, the three spectral exponents log variations exhibit clearly the transition between an homogeneous greenish grey silt and a darker greenish grey laminated silty clay formation as identified by sedimentary observations at 700 m depth. At the bottom of the CSM, the decreasing of electrical resistivities is a result of this accentuated lamination and the proximity of subjacent aquiferous sandstone. Spectral exponent analysis lets one identify all these lithological transitions. In particular, the estimation of spectral exponents reveals the presence of a transition at the very heart of the more homogeneous part of CSM.

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5. Conclusion

We have shown that scaling process model of electrical resistivity measurements can be adapted to describe the multifractal heterogeneities of compacted, cemented and anhydrous siliciclastic unit. Even the silty unit is usually assumed to be perfectly homogeneous; a logical conclusion from these observations suggests that the geological unit contains a range of physical discontinuities that developed as a response to stress. Indeed, These pseudo logs express the local scaling properties of homogeneties of subjacent physical process and can be used to distinguish different electrofacies within a formation. Statistical methods of clustering applied to these pseudo logs optimise the determination of a continuous lithology.

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References